Ion imaging in a high-gradient magnetic guide


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We study a photoionization method to detect and image a narrow beam of cold atoms traveling along a high-gradient two-wire magnetic guide that is continuously on. Ions are accelerated in a compact acceleration region, directed through a drift region several centimeters in length, and detected using a position-sensitive ion detector. The potentials of several electrodes can be varied to adjust the imaging properties. Using ion trajectory simulations as well as experiments, we study the passage of the ions through the detection system, the magnification of the detection system, and the time-of-flight characteristics. © 2010 American Institute of Physics. [doi:10.1063/1.3386583]

I. INTRODUCTION

Laser-cooled atoms in linear atom guides provide an excellent experimental platform for applications that require highly elongated cold atom samples. The applications include continuous evaporative cooling which may lead to a phase- and amplitude-stable atom laser beam. Cw atom lasers could provide coherent, continuous matter waves for atom interferometric devices (for a recent review of atom interferometry see Ref. 2). A coherent atomic matter-wave beam would also be an ideal source for high resolution, diffraction-limited, direct-write, atom-lithographic applications (for a brief review see Ref. 3). There, an approach based on cold atoms would eliminate space-charge-induced Coulomb repulsion seen elsewhere in ion-beam writing techniques. In view of such prospects, we have recently constructed a two-wire magnetic guide and demonstrated that the guide can transport a small-diameter flow of laser-cooled atoms over distances near 2 m. Another recent surge in interest in linear, laser-cooled atom samples stems from the fact that such systems present the possibility of creating one-dimensional spin chains by exciting some of the atoms into high-lying Rydberg levels that interact via the large dipole moments of such atoms. The prospect of conservative trapping of Rydberg atoms in magnetic atom guides has been studied in Refs. 7–9. The magnetic guide used in our work is suited for preparing highly elongated Rydberg-atom samples and magnetically guiding those Rydberg atoms.

The analysis of cold atomic beams in atom guides as well as the readout of atom-interferometric devices requires highly sensitive atom imaging and counting techniques. Methods that allow one to leave the atom guiding fields on during detection are preferable over methods that require one to turn the guide fields off. The most common method for in situ imaging of guided atomic beams is based on fluorescence or absorption imaging on a cycling transition. In the case of guided $^{87}$Rb atoms, the probe laser is tuned to the $5S_{1/2}, F=2$ $\rightarrow 5P_{3/2}, F'=3$ cycling transition (see, for example, Refs. 10 and 11). If the magnetically guided state prior to imaging differs from the lower state of the cycling transition, a repump laser is needed as well. For instance, if the $^{87}$Rb atoms are guided in the state $5S_{1/2}, F=1, m_F=-1$, a repump laser tuned to the $5S_{1/2}, F=1 \rightarrow 5P_{3/2}, F'=2$ transition may be used to optically pump the guided atoms into the lower level of the cycling transition (see, for instance, Ref. 4 and for a similar case with Na atoms see Ref. 12). This type of fluorescence imaging has the advantage of a large photon yield per atom, leading to a high signal to noise ratio. However, the photon yield per atom varies in space and the method produces copious amounts of stray light. The latter is destructive to nearby guided atoms that are not being imaged. The fluorescence produced by the repump laser may be used to generate an “open-transition” fluorescence image. While this open-transition method results in a much smaller signal, it has the advantage of a relatively constant photon yield per atom. Hence, the open-transition method is well suited to generate spatially resolved images that closely match the area density of the guided atomic beam being imaged. It has also been discovered that the open-transition method improves the accuracy of temperature measurements of guided atomic beams. Since the open-transition method produces less scattered light than the cycling transition method, it is less destructive to guided atoms traveling outside the probe region. However, both fluorescence imaging schemes are destructive to the entire atomic beam within the volume being probed.

In certain applications it is only necessary to detect the guided atoms and no spatial image of the atomic distribution is required. In this context, methods have been studied in which higher sensitivities are achieved via the use of optical cavities. While dispersive cavity-based methods are, in principle, capable of detecting small numbers of atoms in a nondestructive fashion, cavity setups are hard to implement in some experiments and highly nontrivial sensitivity variations will occur whenever spatial variations of level shifts.

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due to the guide fields become substantial (as is often the case). A high single-atom fluorescence detection efficiency has been achieved using optical fibers placed near an atom guide.\textsuperscript{17} As this method employs near-resonant cycling transition fluorescence, the above mentioned general limitations of fluorescence methods remain.

In the present work, we describe and characterize a novel atom counting and imaging setup that is based on photoionization and ion imaging. Using two-step laser excitation, guided atoms are ionized. The generated ions are extracted from the magnetic guide and imaged onto a position-sensitive microchannel plate (MCP) detector using an ion lens. Due to the heavy mass of the detected Rb\textsuperscript{+}-ions, the guide magnetic fields can be left on during ion extraction and detection. The strong, inhomogeneous magnetic fields of the atom guide (fields and field gradients of order 200 G and 2 kG/cm, respectively) cause only minor deflections of the ion trajectories. We characterize the atom detection method using experiments and ion trajectory calculations in which guide fields and ion-ion repulsion have been modeled in detail. The ion detection method is equally well suited for experiments on Rydberg atoms prepared in magnetic guides. While optical techniques are generally not useful to detect Rydberg atoms, state-selective electric field ionization can be employed.\textsuperscript{18} With this technique, ions or electrons resulting from electric field ionization are counted and/or imaged using charged-particle detectors.

The paper is organized as follows. In Sec. II we provide a brief overview of the utilized two-wire magnetic atom guide. In Sec. III, the photoionization process and ion imaging setup are described in detail. In Sec. IV, we explain the methods used to simulate the ion trajectories in the detection system. In Sec. V the results of experiments and ion trajectory simulations are presented. In Sec. VI we evaluate the advantages that the ion detection and imaging method may present over optical detection techniques. We also discuss how this method can be easily adapted to allow for time-resolved, state-selective counting and imaging of Rydberg atoms in an atom guide environment.

II. MAGNETIC GUIDE

The atom guide used in this work has been described in detail in Refs. 4 and 13. Briefly, we employ a two-wire magnetic guide with a field gradient up to 2.7 kG/cm and a length of about 2 m. As shown in shown in Fig. 1, a primary vapor-cell pyramidal magneto-optical trap (PMOT) generates a cold beam of \textsuperscript{87}Rb atoms which is picked up by a secondary 2D\textsuperscript{+} moving MOT (MMOT). The MMOT is collinear with the atom guide and is created by the magnetic field of the guide wires and four laser beams.

By providing a suitable differential detuning between the downward and upward MMOT beams, the atoms collected in the MMOT are launched upward into the magnetic guide with an adjustable velocity. The launched atoms are optically pumped into the dark, magnetically guided ground-state sublevel \(|F=1,m_F=-1\) which is largely insensitive to scattering of MMOT light. Hence, the MMOT and the magnetic atom guide can be operated simultaneously, resulting in a continuous, magnetically guided atomic beam. In most of the horizontal portion of the atom guide, the center-to-center separation of the 3.175 mm diameter guide wires which can carry currents up to 300 A is 4.2 mm, leading to the aforementioned field gradient. We have established previously that the atomic beam has a forward velocity adjustable between 1 and 2 m/s, a longitudinal (along the guide axis) temperature of approximately 1 mK, a transverse temperature of approximately 400 \(\mu\)K, a diameter on the order of 400 \(\mu\)m, and a flux in the range of several \(10^7\) s\(^{-1}\). The atoms are guided from the MMOT toward a photoionization/ion detection region that is located about halfway down the guide and to a fluorescence imaging region near the end of the guide (see Fig. 1). The purpose in the present paper is to characterize the performance of the photoionization/ion detection component. The fluorescence imaging device at the end of the guide is used to verify the presence of a guided atomic beam and to optimize the atomic beam flux.

III. PHOTOIONIZATION AND ION DETECTION UNIT

In the photoionization/ion detection region, atoms are excited into the intermediate state \(5P_{3/2}\) via simultaneous application of an optical-pumping pulse \((5S_{1/2}, F=1 \rightarrow 5P_{3/2}, F=2)\) and an excitation pulse on the cycling transition \(5S_{1/2}, F=2 \rightarrow 5P_{3/2}, F=3\). Both of these lower-transition lasers have a diameter larger than that of the guided atomic beam (\(~400\) \(\mu\)m) and an intensity of order ten saturation intensities \((I_{sat}=1.6\text{ mW/cm}^2)\). The two-step photoionization process is completed by a 479 nm laser beam that is always on and that excites the atoms from the \(5P_{3/2}, F=3\), state into the \(eS\) and \(eD\) continua of Rb, where \(e\) denotes the energy of the photoelectron. The upper-transition laser beam is focused to a spot size of \(~120\) \(\mu\)m and has a power of 2 mW. The effective duration of the photoionization is given by the duration of the lower-transition pulse which is 20 \(\mu\)s. To estimate the number of ions produced in one pulse, we assume an atomic beam flux of several \(10^7\) s\(^{-1}\) (Ref. 4) and an atomic beam velocity of 1 m/s, leading to a

![FIG. 1. (Color online) Sketch of the experimental setup. A beam of cold atoms emerging from a primary PMOT is collected in a secondary MMOT which is collinear with the magnetic atom guide. Atoms are continuously launched upwards into the atom guide. Upon exit from the MMOT, the atoms are optically pumped into a magnetically guided dark state that is insensitive to MOT stray light. The guided atomic beam can be analyzed using a fluorescence imaging device near the end of the guide. The photoionization/ion imaging detector studied in this paper is located near the midpoint of the horizontal guide section.](Image)
linear atom density of several $10^7$ m$^{-1}$. Assuming that the lower transition is entirely saturated, the number of $5P_{3/2}$-atoms irradiated during one photoionization pulse is of order 1000. The fluence $F$ of the 479 nm laser over the 20 $\mu$s duration of a single photoionization pulse, estimated to be of the order $F=10^{15}$ cm$^{-2}$ for the above beam parameters, and the near-threshold photoionization cross section of the $5P_{3/2}$ level $\sigma_{PI}=1.4 \times 10^{-17}$ cm$^2$ (Ref. 19) lead to a photoionization probability per atom and pulse of 1–2%. Hence, several tens of ions per photoionization pulse are expected, out of which 30%–50% are registered by the MCP detector. This is what is observed in the experiment described in Sec. V.

The ion detection unit incorporates an extraction electrode (electrode in Fig. 2) mounted 3 mm above the guiding channel of the magnetic guide formed by the two inner wires (guide wires in Fig. 2). In the work reported here, the extraction electrode is held at a variable, negative voltage (set to $-1.4$ kV in Fig. 2). All ions created by photoionization are extracted quasi-instantaneously. A guard tube, held at a constant potential of $-2$ kV, shields the electric field created by the extraction electrode and defines an electric-field-free ion drift region near the MCP. Narrow slits (10 mm long by 1 mm wide) in both the electrode and guard tube allow the ions to pass through and be imaged on the MCP surface. In the following we define the $z$-axis to be parallel to the guide wires and the $x$- and $y$-axes as shown in Fig. 2.

IV. SIMULATION DETAILS

The motion of the ions in this geometry was simulated by first placing a collection of ions in between the guide wires, then using a numerical integrator to calculate their trajectories in the presence of the magnetic fields of the guide, and finally mapping their final positions on the MCP detector. In order to replicate realistic initial conditions, 300 ions were placed according to a cylindrically symmetric Maxwellian thermal distribution with a standard deviation of 50 $\mu$m transverse to the guide axis, corresponding to the approximate diameter of the 479 nm photoionization beam. The initial momentum space distribution was assumed to follow a Maxwellian thermal distribution with a temperature of 400 $\mu$K along the transverse ($x$ and $y$) axes and 1 mK along the guide ($z$) axis.

The equations of motion for the ions were integrated using a fourth-order Runge–Kutta routine with a time step of 1 ns. The magnetic field of the guide wires, the electric field generated by the electrodes, and the electric field due to the ion-ion interactions were all taken into account when calculating the net force on each ion.

The Coulomb force on the $j$th ion due to the other ions was calculated using a pairwise sum over all particles

$$
F_j = \frac{q_j}{4\pi\varepsilon_0} \sum_{i \neq j} \frac{q_i q_j}{r_{ij}^2},
$$

with ion positions $r_i$ and $r_{ij}=r_i-r_j$.

The magnetic field was calculated using the analytical expression for the field of two parallel current carrying wires, given by

$$
B_x = \frac{\mu_0 d}{2\pi} \left( \frac{1}{r_x^2} + \frac{1}{r_x''^2} \right),
$$

$$
B_y = -\frac{\mu_0 d}{2\pi} \left( \frac{x-d/2}{r_x^2} + \frac{x+d/2}{r_x''^2} \right),
$$

where $r_x$ and $r_x''$ are the distances between the ion and the right and left guide wires, respectively, and $d$ is the center-to-center separation of the guide wires.

The electric potential in the region of the guide wires and the electrode was determined using an adaptive finite element algorithm implemented by Ansoft’s “MAXWELL” software. The electric field calculated from the potential was then exported onto a regular two-dimensional square mesh with a spatial resolution of 2.54 $\mu$m. The external electric field due to the electrodes at the position of each ion was interpolated from the mesh. Due to the high resolution of the mesh, which was required to accurately model the ion dynamics near the edges of the electrodes, the overall extent of

![FIG. 2. (Color online) Detailed sketch of the photoionization and ion detection unit to scale. The wire diameter is 3.18 mm with a surface to surface separation of 1.49 mm. The electrode is 1.59 mm thick with a slit width of 1.02 mm. The distance from the center of the guide wires to the top of the electrode is 4.76 mm, and the intermediate region between the electrode and the guard tube is 1.59 mm. The guard tube has a slit width of 1.59 mm. The distance from the top of the guard tube opening to the face of the MCP is 56.2 mm.](Image)
the mesh had to be cut off for practical reasons. This was possible because the electric field in the drift region, inside the guard tube, falls off sharply as a function of distance from the entrance slit. Once inside the drift region, the ions are not subject to a substantial force. We therefore cut off the electric field grid inside the guard tube at a height of about 2 mm above its entrance slit. The electric field at locations above the grid cutoff remained below 10 V/cm which was negligible for the present purpose. The effect of the magnetic field was also negligible in this region. Therefore, once an ion reached the grid cutoff height, the final ion image location on the MCP was obtained in one step by linear extrapolation of the ion trajectory from the grid cutoff plane onto the MCP front plane.

V. RESULTS

A. Ion trajectories

Using the above described method, we have computed ensembles of ion trajectories for different (fixed) voltages applied to the extraction electrode. In the simulations, the MCP front plate and the guard tube are held at a fixed potential of −2 kV. To illustrate the focusing and magnification behavior of the ion imaging, the initial ion ensemble is spread out along the x-direction over a range of 100 μm, corresponding to typical experimental conditions. A typical simulation result is shown in Fig. 3.

The slits in the guide rail, electrode, and guard tube introduce a significant electric field component along the x-direction in the guiding and intermediate regions. The x-component of the field causes the width of the ion distribution along x to change as the distribution travels toward the MCP during extraction. Regions where the width of the ion distribution reach a minimum correspond to the focal spots of the ion imaging system. The positions of the focal spots along the y-axis can be varied by changing the potentials of the electrode. Consequently, the setup behaves like a microscope with an adjustable magnification factor in the x-direction. For the potentials used in Fig. 3, there are two intermediate focal spots, as can be clearly seen in the figure, and the x-magnification factor is about 15. Note that the MCP position is off the y-axis in Fig. 3; hence, the x-magnification factor is larger than it appears. Due to the (approximate) translational symmetry of the system in the z-direction, the z-magnification factor is unity regardless of the detailed conditions.

Before presenting details in the next two subsections, we report some general findings. As the guide-wire magnetic field has to be left on during ion extraction and detection, it is not possible to detect electrons (as might be done by reversing the imaging voltages). Over the relevant energy and magnetic field range, the electron cyclotron radii are so small that in most spatial regions the electrons exhibit drift trajectories and largely follow the magnetic field lines of the guide-wire field. As a result, electrons generated by field ionization are channeled away from the electrode slit and the MCP detector. Due to the heavy mass of Rb ions, the ion trajectories are only slightly deflected by the magnetic field. Since the ions have velocities that mostly point in the +y-direction, and since the guide magnetic field along the ion trajectories mostly points in the +x-direction, the Lorentz force results in an ion deflection in the −z-direction. Under typical conditions, the ion deflection only amounts to several 100 μm, as seen in Fig. 4, which shows top-down views of initial and imaged distributions under typical conditions. The ion imaging characteristics along the x-direction are not noticeably affected by the guide magnetic field.

Using simulations of simultaneous extraction and detection of several hundreds of ions, we have evaluated the effect of the Coulomb repulsion between the ions. The repulsion is potentially significant during the early stages of extraction...
when the ions are slow and close to each other and at times when the ions pass through the focal spots. The results show that over the experimentally relevant range of the ion density the ion imaging is not adversely affected by Coulomb repulsion.

B. The magnification factor

The x-magnification factor of the ion distribution $M$ is closely related to the position of the second focal spot along the $y$-axis and can be varied by altering the electrode potential. In Fig. 5, we show $M$ versus the extraction electrode potential for a constant guard tube potential of $-2$ kV. The figure clearly shows that there are two distinct regimes where the magnification behaves differently. For low absolute values of the extraction electrode potential, there are two focal spots, as seen in Fig. 3. The first focal spot, which is present under all conditions studied, occurs near the origin of the ion ensemble, at a height near $y=1$ mm. The second focal spot forms within the range of the guard tube. As the absolute values of the extraction electrode voltage are increased, the second focal spot approaches the MCP and consequently $M$ decreases. At approximately $-500$ V, the second focal spot forms on the surface of the MCP, resulting in a very small magnification. For extraction electrode voltages more negative than $-500$ V, the second focal spot effectively forms above the MCP and therefore acts as a virtual focal spot of the ion imaging device. The value of $M$ begins to increase again and reaches approximately 15 at an extraction electrode potential of $-1500$ V. It is further noted that for voltages more negative than $-500$ V the image is inverted along the $x$-axis whereas for voltages between 0 and $-500$ V it is not. In Fig. 5 we show experimentally obtained ion images in the various regimes, confirming the behavior of the $x$-magnification factor found in the simulations. Also, in the displayed experimental images a small field asymmetry has lead to an offset of the average $x$-position of the detected ions. A comparison of the average $x$-positions in the regime of two focal spots (left picture) and the regime of one focal spot (right picture) with the case that the second focal spot lies in the MCP plane (middle picture) confirms the change in $x$-inversion behavior.

The transverse magnification factor is dependent on the $y$-height of the initial ion distribution relative to the plane of the guide wires. The effect of initial height $y_0$ was investigated in simulations by choosing initial ion distributions of the type $P(x,y) = g(x)\delta(y-y_0)$, with a Gaussian $g(x)$ and varied values of the initial height $y_0$. Figure 6 shows an approximately linear relation between $y_0$ and the magnification factor. The significance of the $y_0$-dependence of $M$ in experiments is largely given by the vertical width of the initial ion distribution which is of the order of the diameter of the laser beams used for photoionization. For a laser beam size of 25 $\mu$m, which is fairly straightforward to realize, Fig. 6 indicates a variation of $M$ over a range $13.4 \leq M \leq 13.8$ (i.e., a 3% variation). This variation is small enough that it would not compromise most applications.

C. Times-of-flight

In applications of the ion imaging device, the average time-of-flight of ions (i.e., the time elapsed between photoionization and arrival at the MCP image plane) will be important. Due to the three-dimensional character of the ion distributions, we expect some dispersion of the times of flight; estimates of the time-of-flight dispersion will also be important. To study the time-of-flight characteristics, we use simulated initial ion distributions with Gaussian profiles in the $x$- and $y$-directions with a standard deviation of 100 $\mu$m and a uniform distribution along the $z$-axis. Several simulated time-of-flight distributions are shown in Fig. 7. Typical arrival times of the ions are on the order of 1 $\mu$s with up to several 100 ns of dispersion. Ions closer to the electrode arrive earlier.

As one would expect, increasing the magnitude of the potential on the electrode reduces the time-of-flight of the ions to the MCP due to the increased initial acceleration of ions in the guiding region. For increasing magnitude of the extraction voltage, the average time-of-flight approaches an
asymptotic value of 0.87 \mu s (for the given geometry). In the limit of very large, negative extraction voltage (\leq -2 kV), the extraction electrode generates a very deep potential well for the ions that pass through it. The time-of-flight through the well has an asymptotic value of 0. Regardless of the detailed shape of that well, the ions slow back down to a kinetic energy of 2 kV while exiting the well and entering the guard tube (neglecting initial variations of potential energy). The time-of-flight is then mostly given by the 2 kV kinetic energy and the length of the tube, giving rise to the observed asymptotic behavior. For lower absolute values of the extraction voltage, the time the ions spend between the photoionization point and the guard tube, which adds to the asymptotic time-of-flight value, becomes significant and dominates the time-of-flight for extraction voltages between about -100 and 0 V. Also, at low absolute values of the extraction voltage the dispersion becomes more significant due to an increased effect of variations in initial conditions.

VI. CONCLUSION AND DISCUSSION

We have proposed and demonstrated a method to image atomic beams in a magnetic guide using photoionization and ion detection. This method can be used as an alternative to fluorescence imaging methods, which rely on a resonant probe beam illuminating the guided atoms (usually the 5S – 5P transition for the case of Rb). Unlike fluorescence imaging, where the probe beam usually causes near-complete atom loss from the atom guide due to spontaneous decay of the 5P state, photoionization can be performed with vanishing spontaneous scattering and the associated atom loss effects. This can be achieved, for instance, by photoionization of the atoms in one step using UV light (\lambda < 297 nm for Rb). Alternatively, one may use a two-photon photoionization process that is far-off-resonant from any intermediate level. Two useful modes of operation are anticipated:

1. A weak-measurement mode in which a guided atomic beam is monitored continuously by ionizing and detecting a small percentage of the atoms while letting the main fraction of the beam pass in an unperturbed manner. Since the laser wavelength(s) can be chosen far-off-resonant from intermediate levels, atom loss due to photon scattering can be suppressed to any desired degree. This mode is nondestructive in a sense that all atoms that do not contribute to the signal are not affected.

2. A strong-measurement mode in which a high-power far-off-resonant photoionization beam is employed to achieve near-uniform, close to unity ionization efficiency per atom. Accounting for the MCP ion detection efficiency, an overall atom detection efficiency of 30% may be expected. While this is lower than in some optical fluorescence methods, the detection efficiency will be uniform over the entire detection volume and unaffected by guide fields. As in (1), atom loss due to photon scattering can be suppressed to any desired degree by choosing suitable laser wavelength(s). Hence, this mode of operation would enable high-efficiency, continuous atom counting and imaging in guided-atom interferometers.

An important parameter of an atom detection scheme is the range of detectable atom fluxes. In the mode (2) discussed in the previous paragraph, the atom detection efficiency would be of order 30% while typical MCP dark-count rates are of order one per second. Since the dark-count rate could be reduced by spatial gating and detector cooling, we expect a lower limit of the range of detectable atom fluxes below 1 s\(^{-1}\). The detection could be continuous or pulsed. Each detected ion produces a spatially and temporally resolved signal pulse. These features compare favorably with optical methods. An upper limit of the detectable flux range is imposed by Coulomb repulsion forces between the extracted ions. Coulomb interaction effects depend on the photoionization rate, the detection volume, and the electric field used for extraction. Under the conditions studied in this paper, several hundred ions could be extracted over a duration of 1 \mu s without obvious ion-ion repulsion effects, corresponding to a flux of order \(10^8\) s\(^{-1}\). An MCP ion detector could handle fluxes that high if it were operated at a reduced gain. In other ion detection experiments, we have seen clear Coulomb expansion effects at photoionization rates of order \(10^4\) s\(^{-1}\). These figures show that in most applications the upper limit of the range of detectable atom fluxes will be irrelevant.

An attractive aspect of the photoionization and ion detection device studied in this paper is that it can be easily adapted to detect Rydberg atoms in atom guides. In such applications, the potential on the extraction electrode is held near ground most of the time. The photoexcitation of Rydberg atoms and experiments on these atoms would occur under electric-field-free conditions, over a time window ranging up to several milliseconds. To detect and image the Rydberg atoms, the extraction electrode is pulsed to a variable negative voltage, and the ions resulting from electric-field ionization of the Rydberg atoms would be extracted and imaged as explained in this paper. Experiments along these lines are already in progress.

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